Homework 12: Controllability and State-feedback Control

Note: This homework assignment is due on Tuesday 22.05.2012, 13:40.

Problem 27:

We consider a plant with the following state equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_2 & 0 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 0 \\ b_1 \\ 0 \\ b_2 \end{bmatrix} \cdot u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \cdot x(t)$$

Check controllability of the system for the following choices of the parameters a_1, a_2, b_1, b_2.

	a_1	a_2	b_1	b_2
Case 1	-9	25	17	-9
Case 2	0	20	15	0
Case 3	9	25	17	9
Case 4	0	20	15	0

Problem 28:

We revisit the active vehicle suspension system from Problem 1 and 4. We recall the state equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\dfrac{c}{m} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \dfrac{K \cdot A}{m} \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

We assume that all parameters c, K, A, m are positive constants.

a. Show that the system is instable.

We want to stabilize the system by an appropriate state feedback

b. First check that the system is controllable.

c. Now design a feedback vector k such that the poles of the closed-loop system lie at $s_1 = -5$ and $s_2 = -5$.

d. Find the pre-filter value M such that a reference unit step $r(t) = \sigma(t)$ leads to a final output value of $\lim_{t\to\infty} y(t) = 1$

e. Write down the overall state feedback equation. Which states have to be measured?