Homework 3: Linear and Nonlinear State Space Models

Note: This homework assignment has to be returned until Thursday 08.03.2012, 15:40.

Problem 5:

We recall the block diagram for the magnetic suspension system in Problem 1 (Homework 1) and assume that $x_0 = 0$, $v_0 = 0$.

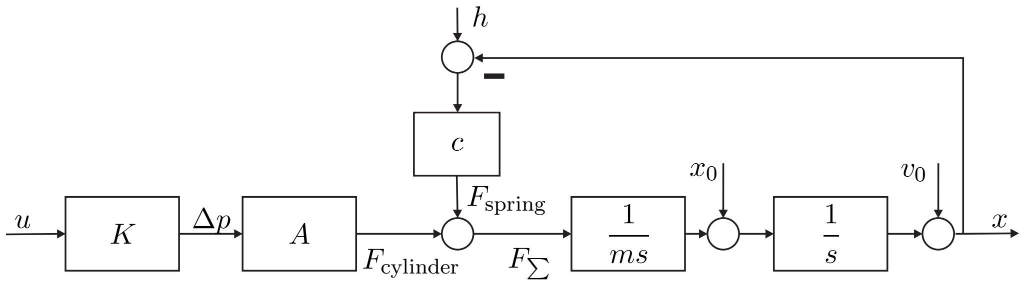


Figure 1: Block diagram of the vehicle suspension system

a. Determine a state space model of the magnetic suspension system

Hint: You can either use the block diagram in Figure 1 or the equations derived in Problem 1 **b.**

b. Compute the transfer functions $G_2(s)$ and $G_1(s)$ in the simplified block diagram from the state space representation.

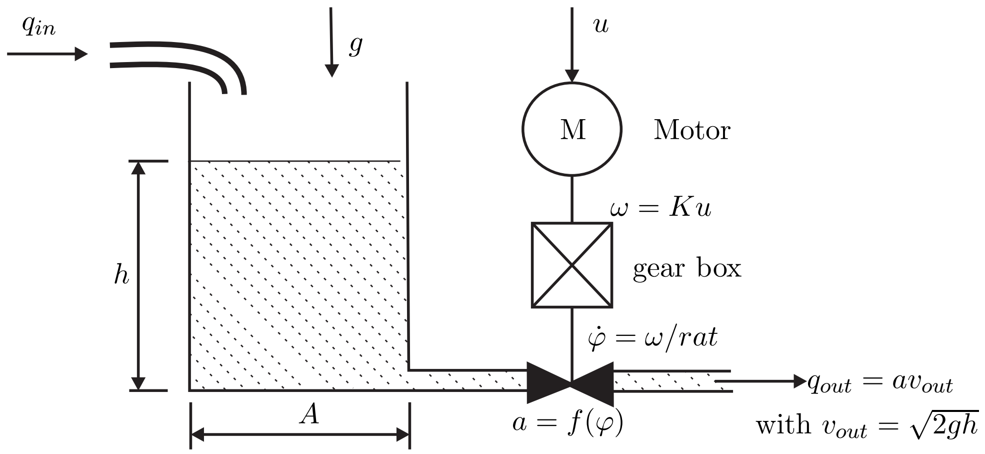
Problem 6:

We are given the cylindric tank system with the surface area A that is shown in the following figure. It is filled with water up to a height h. For control purposes, it is possible to influence h by changing the position of a valve that regulates the outflow q_{out} of the tank. There is no possibility to influence the inflow q_{in}.

The actuation of the valve is performed by a motor that turns with an angular velocity ω depending on the voltage u. The relation between ω and u is described as $\omega = Ku$, whereby K is a constant. In addition, a gear box translates ω into a smaller angular velocity $\dot{\varphi}$ depending on the gear box ratio rat. That is, $\dot{\varphi} = \omega/rat$. The outflow q_{out} can be written as the product of the valve area a and the outflow velocity v_{out}, that is $q_{out} = a \cdot v_{out}$. There is a nonlinear dependency between a and the angle φ that is given by the characteristic curve below. In addition, there is a nonlinear dependency between the outflow velocity v_{out} and the water height h that is given by $v_{out} = \sqrt{2\,g\,h}$ (g is the gravitational acceleration).

a. Develop a nonlinear state space model that describes the dynamic relationship between the input voltage u, the inflow q_{in}, the outflow q_{out} and the water level height h.

Hint: Use h and φ as state variables. First determine how the volume change in the cylinder $\dot{h} \cdot A$ depends on q_{in} and q_{out}.



From now on, we will use the parameter values $K = 7.5\,\mathrm{rad/(sec{\cdot}V)}$, $g = 10\,\mathrm{m/sec^2}$, A $= 150\,\mathrm{cm^2}$, $rat = 15$.

b. Determine the set-point of the tank system for the case that $q_{in,SP} = 50\,\mathrm{cm^3/sec}$ and $\varphi_{SP} = 4\pi$.

Hint: You should the characteristic curve in the figure below to find the set-point value of the valve surface a_{SP}.

c. Linearize the system around this set-point and write down the linear state space model

d. Compute the transfer function between the input u and the output h. What is the type of the transfer function?