Homework 4: Properties of LTI Models

Note: This homework assignment is due on Thursday 15.03.2012, 15:40.

Problem 7:

The following pole-zero representations of two transfer functions G_1 and G_2 with the constant coefficient $K = -2$ are given in the following figure.

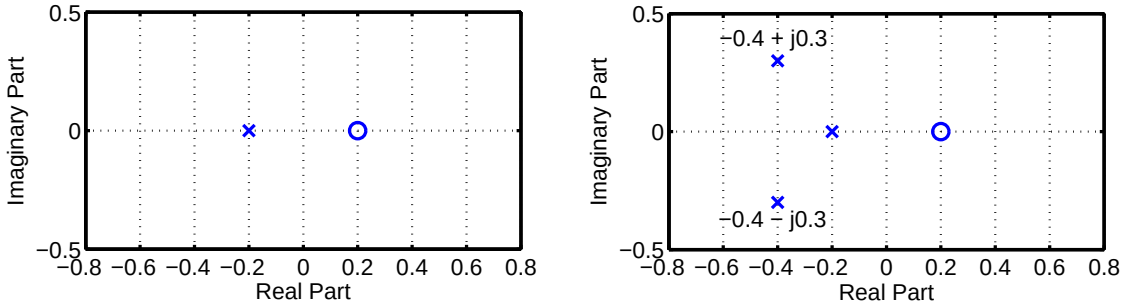

Figure 1: Pole-zero representation of the transfer functions (a) G_1 and (b) G_2

a. Write down the transfer functions G_1 and G_2 in pole-zero representation.

b. What is the relative degree of G_1 and G_2?

c. What can you say about the stability of G_1 and G_2?

d. Assume G_2 was determined from a state space model. Which of the following matrices could be the system matrix of this state space model?

$$A_1 = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.8 & -0.5 \\ 0 & 0.5 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.8 \end{bmatrix} \qquad A_3 = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.4 & 0 \\ 0 & 0 & -0.3 \end{bmatrix}$$

Problem 8:

The following transfer functions are given.

$$G_1(s) = \frac{s^4 + 3s + 2}{s^2 + 4s - 3} \qquad G_2(s) = \frac{5\,s + 20}{(s-4)(s^2 + 2s + 4)} \qquad G_3(s) = \frac{s - 4}{(s+4)(s+7)}$$

a. Which of the transfer functions is proper? Justify your answer.

b. Which of the transfer functions is stable? Justify your answer.

c. Apply the Routh-Hurwitz method to all transfer functions.

d. Write down the pole-zero representation of $G_2(s)$.