ECE 488 – Automatic Control

Bode Plot Design

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Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Weeks

- LTI system modeling
- Nonlinear modeling and linearization
- Stability
- Steady-state and transient response
- Feedback Control
  - Root locus
  - Nyquist plot
- Bode plot

This week

- Gain and Phase Margin
- Lead-lag compensation

Stability Margins: Gain Margin

Assumption

- Open loop transfer function $G_o(s)$ without poles in the open right half plane

Gain Margin

- Multiplication of G_o with constant K_g leads to instable closed loop
- Phase crossover frequency ω_p such that $\angle G_o(\omega_p) = -\pi$
  $\rightarrow$ Gain margin K_g describes degree of stability with respect to gain changes

Illustration

Gap 1

Stability Margins: Phase Margin

Assumption

- Open loop transfer function $G_o(s)$ without poles in the open right half plane

Phase Margin

- Multiplication of G_o with $e^{-j\Phi_m}$ (phase shift of Φ_m) leads to instable closed loop
- Gain crossover frequency ω_g such that $|G_o(\omega_g)| = 1$
  $\rightarrow$ Phase margin Φ_m describes degree of stability with respect to phase shift

Illustration

Gap 2

Stability Margins: Bode Plot

Relation between Nyquist Plot and Bode Plot

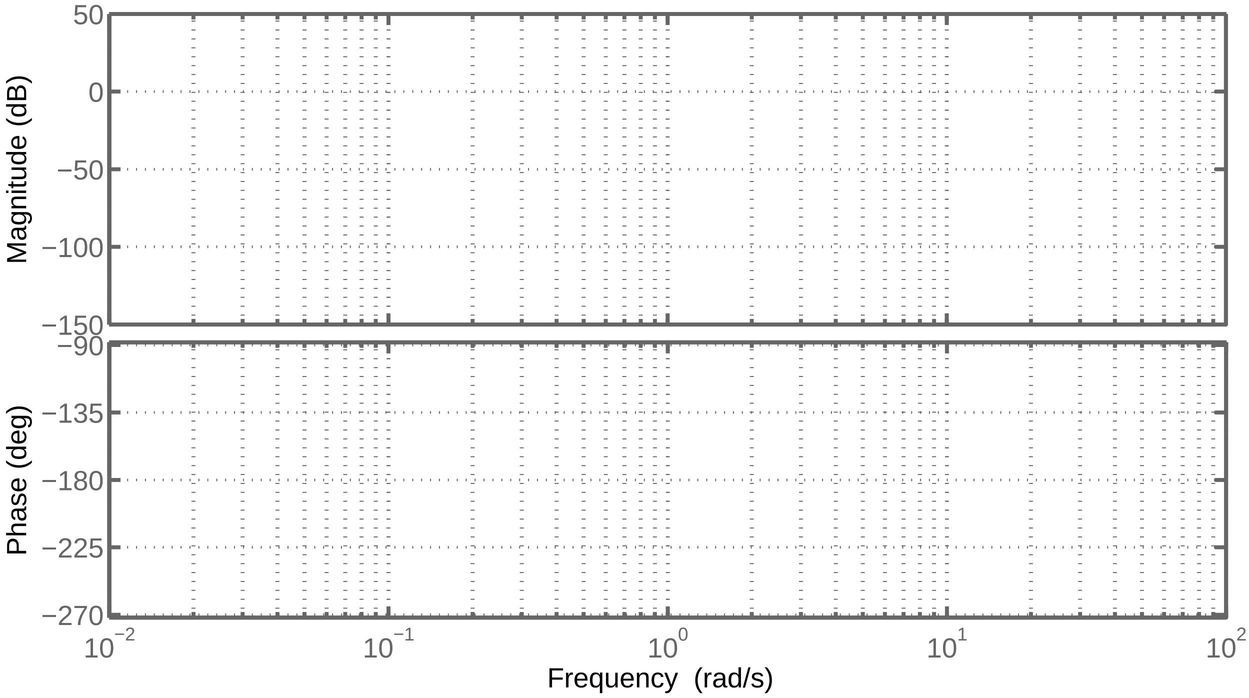
- Magnitude plot: distance of Nyquist curve from origin for $s = j\omega$
- Phase plot: phase angle of Nyquist curve for $s = j\omega$
- Gain crossover frequency ω_G: intersection of magnitude plot with 0-dB line
- Phase crossover frequency ω_P: intersection of phase plot with $-180°$

Stability Condition

- Assumption: $G_o(s)$ has no poles in the ORHP
- Condition: Closed loop is internally stable if
  - phase margin $\Phi_p > 0$
  - phase between $0 < \omega < \omega_G$ is larger than $-540°$

Stability Margins: Example

Bode Plot



Stability Margins: Example

Nyquist Plot

Gap 3

Bode Plot Design: Dominant Complex Pole Pair

Approximated Closed-loop Transfer Function

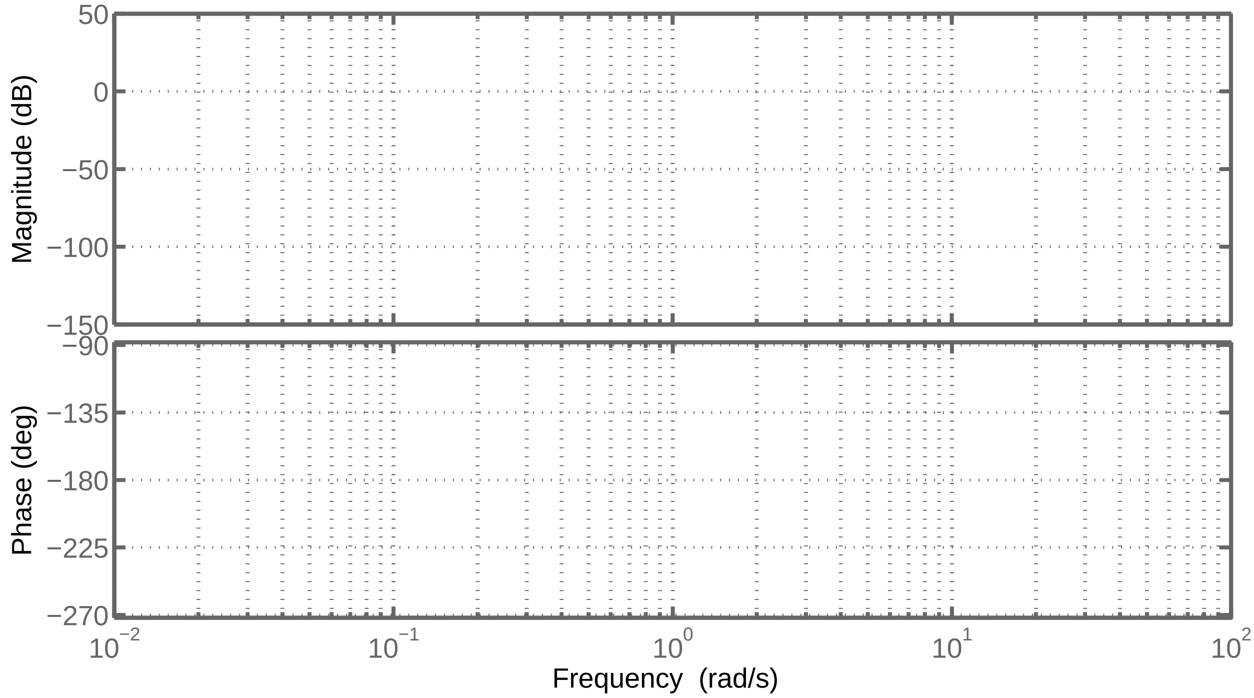
$$T(s) = \frac{K}{1 + 2\,D\,T\,s + T^2\,s^2}$$

Performance Parameters

- DC Gain: $K_{DC} = K$
  $\Rightarrow$ Determines final value of the unit step response
- Phase margin: $\Phi_m = \arctan\left(\frac{2\,D}{\sqrt{-2\,D^2 + \sqrt{1 + 4\,D^4}}}\right)$
  $\Rightarrow$ Determines amplitude of oscillations and relative stability
- Cut-off frequency: $\omega_c = \frac{1}{T}\sqrt{1 - 2\,D^2 + \sqrt{(1 - 2\,D^2)^2 + 1}}$
  $\Rightarrow$ Frequency, where $|T(j\omega)|_{dB}$ is $3\,dB$ below its maximum value
  $\Rightarrow$ Determines speed of response

Closed-loop Requirements: Example

Bode Plot of $T(s)$



Closed-loop Requirements: Performance Specification

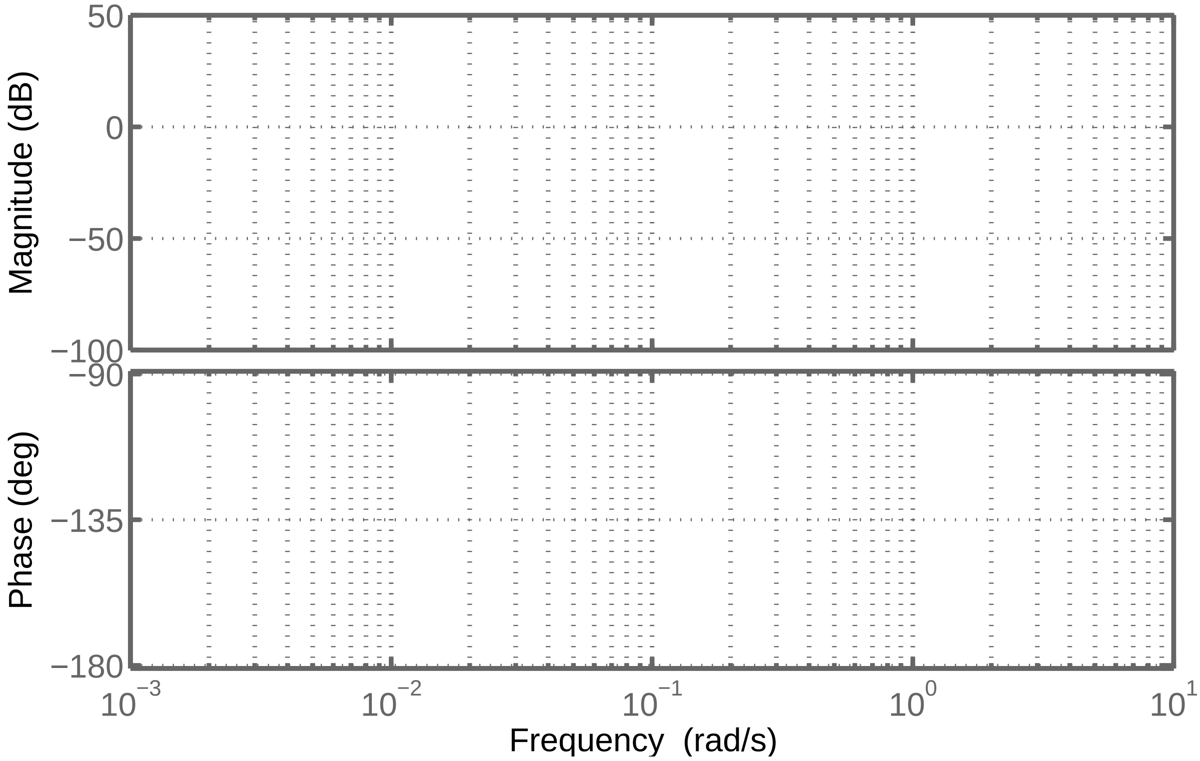
Desired Performance

- DC Gain: $K \approx 1$

Gap 4

- Open-loop transfer function: $G_o(s) = \dfrac{1}{2\,D\,T}\dfrac{1}{s\,(1+\frac{T}{2\,D}\,s)}$
- Sufficient relative stability and small overshoot:
  $\Rightarrow$ Appropriate choice of D. Generally, $D > 0.4$
  $\Rightarrow$ Slope of $|G_o(j\omega)|_{dB} \approx 20\,dB/\text{decade}$
- Sufficient speed of response:
  $\Rightarrow$ Appropriate choice of ω_c

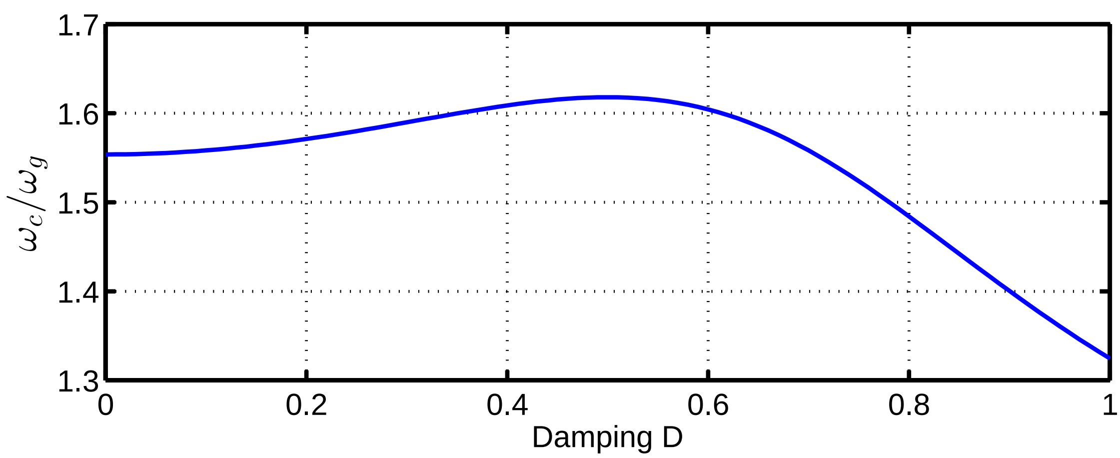
Frequency Response: Open-loop Bode Plot



Closed-loop Requirements: Open-loop Transfer Function

Parameters in Bode Diagram

- Gain crossover frequency: $\omega_g = \frac{1}{T}\sqrt{-2\,D^2 + \sqrt{1+4\,D^4}}$
  $\Rightarrow$ It holds that $\omega_g \approx \omega_c$



- Phase margin: Φ_m
  $\Rightarrow$ Compute from stability requirement. Usually, $\Phi_m > 40^\circ$

Closed-loop Requirements: Basic Control Design

Desired Open Loop Transfer Function

- Determine the closed-loop requirements
  $\Rightarrow \omega_c$ and Φ_m
- Determine the desired open-loop transfer function $G_o(s)$
  $\Rightarrow \omega_g \approx \omega_c$ and Φ_m
  $\Rightarrow$ Slope of $|G_o(j\omega)|_{dB} \approx 20\,dB/$decade around ω_g

Adjust Real Open Loop Transfer Function

- Determine the Bode plot of the plant transfer function $G(s)$
  $\Rightarrow$ Fixed part of the control system
- Design the controller transfer function $C(s)$.
  $\Rightarrow$ Choose the controller poles, zeros and gain such that Bode plot of $C(s)\,G(s)$ approximately matches the desired $G_o(s)$ around the frequency ω_g

Lead Compensator: Transfer Function

Time-constant Representation

$$C(s) = K_\alpha \,\frac{1 + T\,s}{1 + \alpha\,T\,s}$$

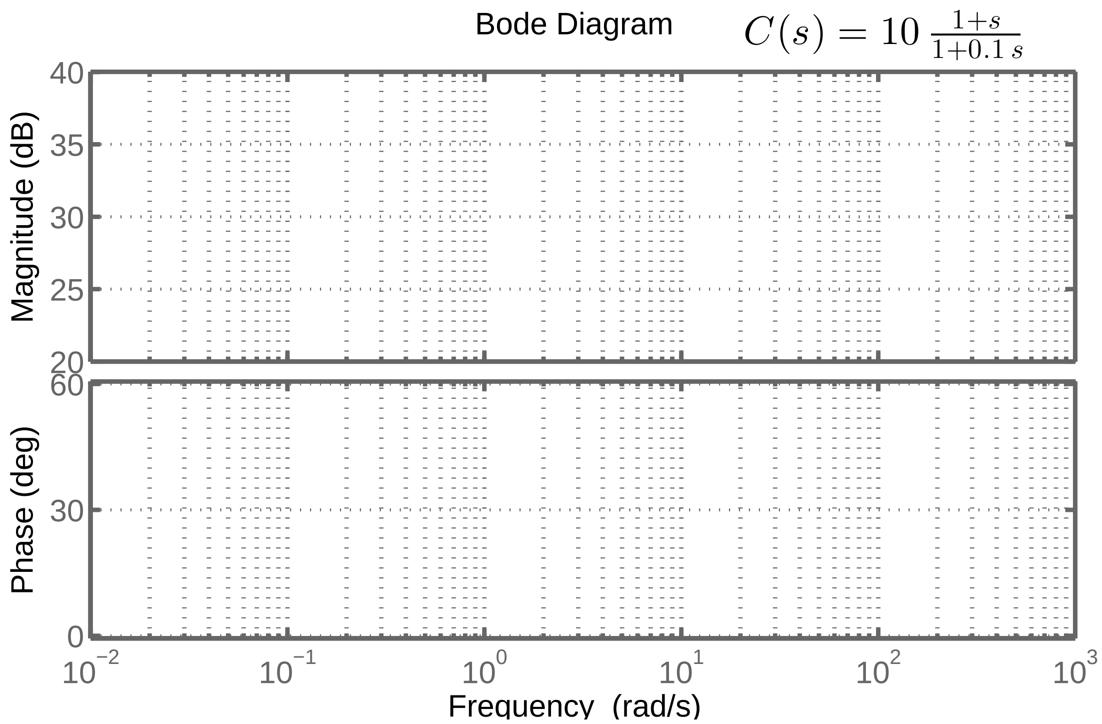
Explanation

- Attenuation factor $0 < \alpha < 1$
- Gain K_α
- Pole at $s = -\frac{1}{\alpha\,T}$
- Zero at $s = -\frac{1}{T}$

Remarks

- Phase increase (lead) up to a maximum value of $\sin(\varphi_\alpha) = \frac{1-\alpha}{1+\alpha}$
- Frequency of maximum phase lead at $\omega_\alpha = \frac{1}{\sqrt{\alpha}\,T}$
- Magnitude at ω_α is $|C(j\omega_\alpha)| = \frac{1}{\sqrt{\alpha}} > 1$

Lead Compensator: Bode Plot



Lead Compensator: Usage

Goal

- Reshape frequency response curve to give additional phase lead in order to increase the phase margin

Starting Point

- Plant transfer function $G(s)$
- Lead compensator transfer function $C(s) = K_\alpha \dfrac{1 + T\,s}{1 + \alpha\,T\,s}$
- Desired phase margin Φ_m
- Steady-state error e_∞

Task

- Determine the parameters K_α, α and T

Lead Compensator: Procedure

1. Determine the gain K_α to achieve the static error specification
2. Draw a Bode plot of $K_\alpha\, G(j\omega)$ and determine the phase margin Φ'_m
3. Determine the required lead angle $\varphi_\alpha = \Phi_m - \Phi'_m + 10^\circ$
   $\Rightarrow \alpha = \dfrac{1-\sin(\varphi_\alpha)}{1+\sin(\varphi_\alpha)}$
4. Choose the gain crossover frequency ω_g such that
   $|K_\alpha\, G(j\omega_g)|_{dB} = -20\log(\frac{1}{\sqrt{\alpha}})$
   $\Rightarrow$ Choose $\omega_\alpha = \omega_g$
5. Evaluate $\omega_\alpha = \frac{1}{\sqrt{\alpha}\, T}$
   $\Rightarrow T = \dfrac{1}{\omega_\alpha \sqrt{\alpha}}$
6. Verify if the design fulfills the specified requirements. Go back to step 3. if the requirements are not fulfilled

Lead Compensator: Example

Computation

Gap 5

Lead Compensator: Example

Computation

Gap 6

Lead Compensator: Example