ECE 488 – Automatic Control

PID Control

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Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Weeks

- LTI system modeling
- Nonlinear modeling and linearization
- Stability
- Steady-state and transient response
- Feedback Control
    - Root locus
    - Nyquist plot
    - Bode plot and Lead/lag compensation

This week

- Lag compensator
- PID controller
- PID design

Lag Compensator: Transfer Function

Time-constant Representation

$$C(s) = K_\beta \, \frac{1 + T\,s}{1 + \beta\,T\,s}$$

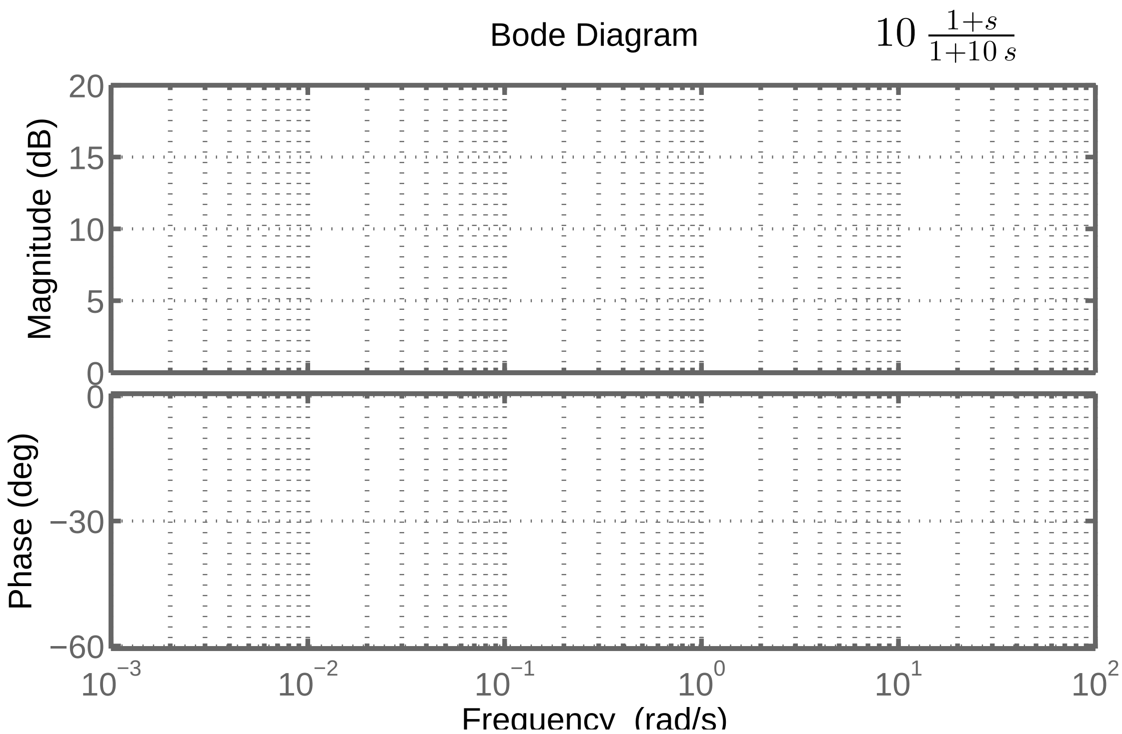
Explanation

- Attenuation factor $\beta > 1$
- Gain K_β
- Pole at $s = -\frac{1}{\beta\,T}$
- Zero at $s = -\frac{1}{T}$

Remarks

- Phase decrease (lag) up to a maximum value of $\sin(\varphi_\beta) = \frac{\beta-1}{\beta+1}$
- Magnitude difference between low frequencies and high frequencies: $-20\,\log(\beta)$

Lag Compensator: Bode Plot



Lag Compensator: Usage

Goal

- Reshape frequency response curve to give attenuation in the high-frequency range in order to provide enough phase margin

Starting Point

- Plant transfer function $G(s)$
- Lag compensator transfer function $C(s) = K_\beta \frac{1 + T\,s}{1 + \beta\,T\,s}$
- Desired phase margin Φ_m
- Static position/velocity error e_∞

Task

- Determine the parameters K_β, β and T

Reminder Lag Compensator Lag-Lead Compensator PID Controller Ziegler-Nichols

Lag Compensator: Procedure

1. Determine the gain K_β to achieve the static error specification
2. Draw a Bode plot of $K_\beta\,G(j\omega)$
3. Find the gain crossover frequency ω_g at the point where the phase angle is equal to $-180^\circ + \Phi_m + 10^\circ$
4. Choose $\frac{1}{T}$ one octave (factor 2) or better one decade (factor 10) below ω_g
   $\Rightarrow$ Phase lag does not affect gain crossover frequency
5. Determine β such that $20\log(\beta) = |K_\beta\,G(j\omega_g)|_{dB}$
   $\Rightarrow$ Lag compensator decreases magnitude curve such that it crosses the 0-dB line at ω_g
6. Verify if the design fulfills the specified requirements. Go back to step 3. if the requirements are not fulfilled

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Lag Compensator: Example

Computation

Gap 1

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Lag Compensator: Example

Computation

Gap 2

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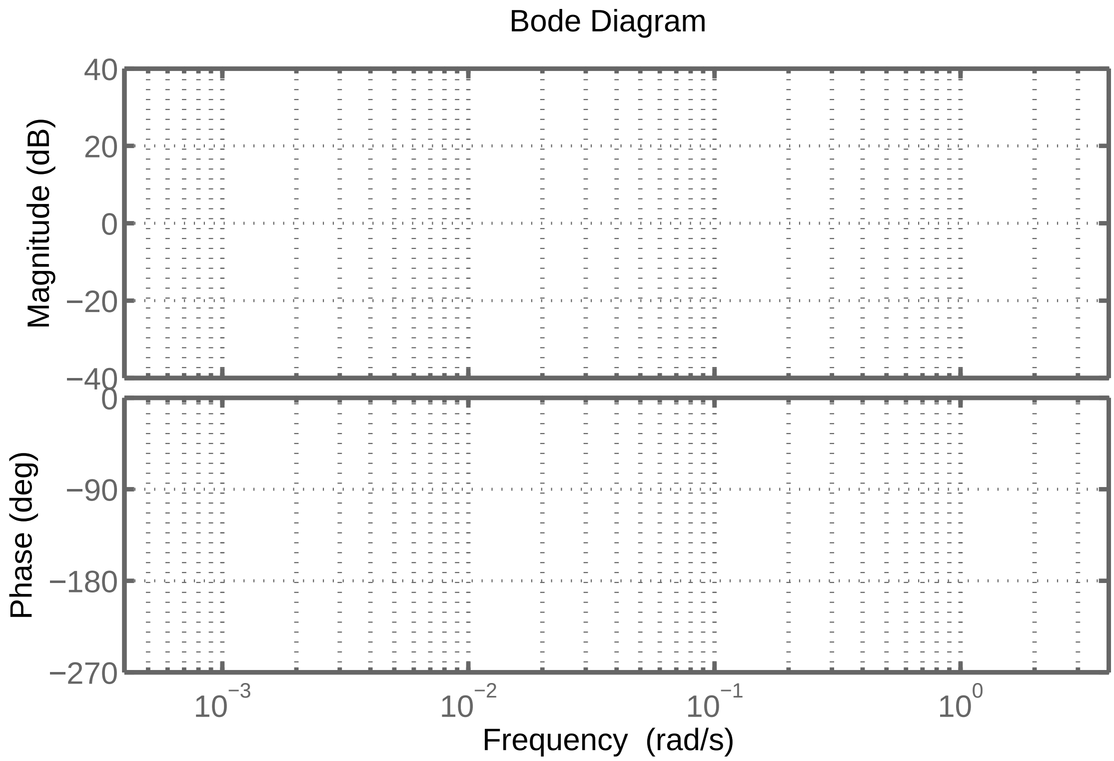
Lag Compensator: Example



Lag-Lead Compensator: Explanation

Transfer Function

$$C(s) = K\,\frac{1 + T\,s}{1 + \alpha\,T\,s}\,\frac{1 + T\,s}{1 + \beta\,T\,s}$$

- $0 < \alpha < 1$ and $\beta > 1$

Description

- Combines advantages of lead compensator and lag compensator
- No detailed discussion in this lecture
- See for example textbook by Ogata, Section 9-4

PID Controller: Characteristics

Gap 3

Ordinary Differential Equation (ODE)

$$u = K_p \cdot (e + \frac{1}{T_I} \int e + T_D \dot{e})$$

Transfer Function (TF)

$$U(s) = K_p \cdot (E(s) + \frac{1}{T_I\, s} E(s) + T_D\, s\, E(s) = K_p\, (1 + \frac{1}{T_I\, s} + T_D\, s)\, E(s)$$

PID Controller: Parameters

Proportional Action: $K_p \cdot e$

- Depends on instantaneous value of error
- Can control any stable plant but usually with low performance

Integral Action: $\frac{K_p}{T_I} \cdot \int e$

- Realizes memory due to dependency on accumulated error
- Enforces steady state error of $\lim_{t \to \infty} e(t) = 0$

Derivative Action: $K_p T_D \cdot \dot{e}$

- Captures trend of the error due to dependency on rate of change of e
- Susceptible to amplification of high-frequency disturbances/noise

PID Controller: Parameters

Illustration

Gap 4

PID Controller: Special Cases

P-Controller

$$C(s) = K_p$$

PI-Controller

$$C(s) = K_p\,(1 + \frac{1}{T_I\,s})$$

PD-Controller

$$C(s) = K_p\,(1 + T_D\,s)$$

Design Task

- Determine the most suitable controller type and the controller parameters K_p, T_I and T_D in order to fulfill given performance specifications

Ziegler-Nichols: Oscillation Method

Assumption

- Stable, non-oscillatory plant: $G(s) = K \dfrac{e^{-s\tau}}{(1+sT_1)\cdots(1+sT_n)}$
  excluding first-order/second-order lag
- Note: plant is not modeled!

Practical Experiment

- Start with $K_p = 0$ and increase K_p gradually until y oscillates
  $\Rightarrow$ Critical gain K_{crit}
- Note oscillation period T_{crit}

Control Loop with P-control

Gap 5

Ziegler-Nichols: Oscillation Method

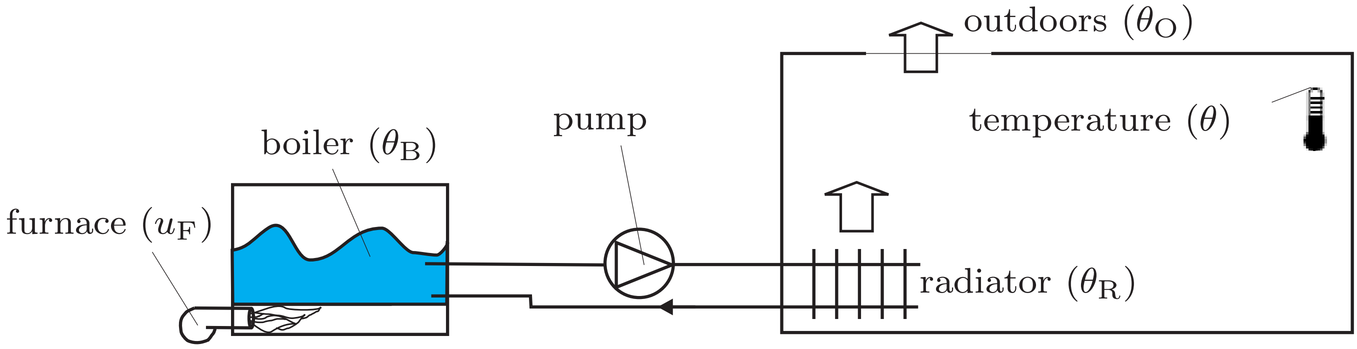
PID-controller Parameters

Controller	K_p	T_I	T_D
P-	$0.5K_{\mathrm{crit}}$	∞	0
PI-	$0.45K_{\mathrm{crit}}$	$0.85T_{\mathrm{crit}}$	0
PID-	$0.6K_{\mathrm{crit}}$	$0.5T_{\mathrm{crit}}$	$0.12T_{\mathrm{crit}}$

Results

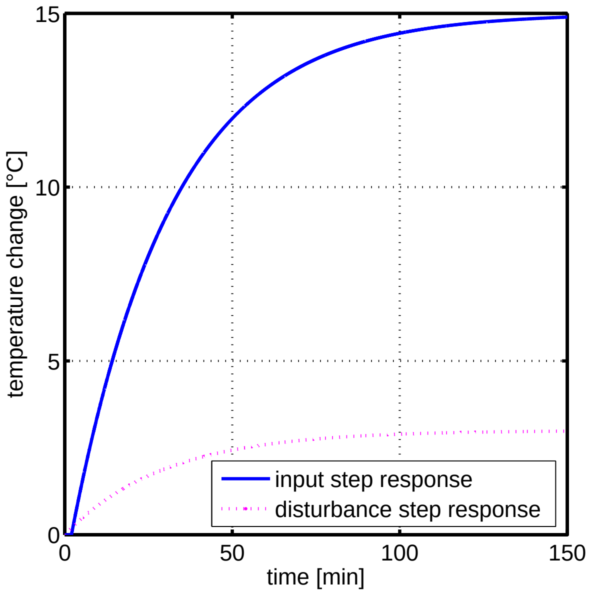
- Stable closed loop
- Addresses both reference tracking and disturbance rejection

Example (temperature control)

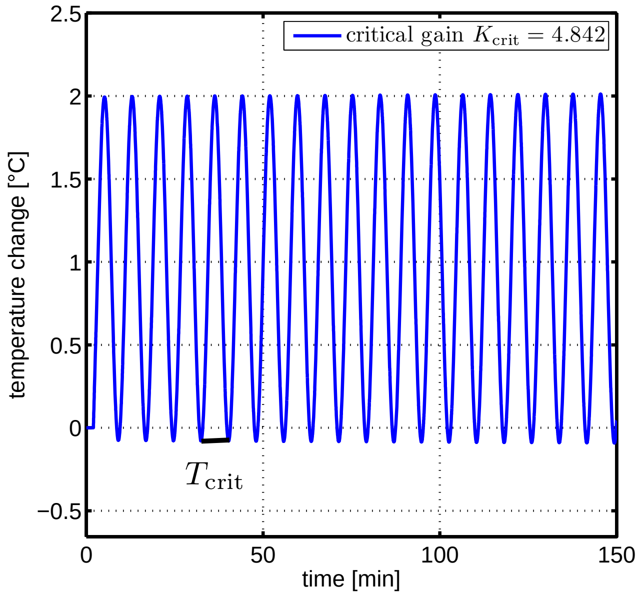


Ziegler-Nichols: Oscillation Method

Uncontrolled Plant Step Response



Oscillation Experiment



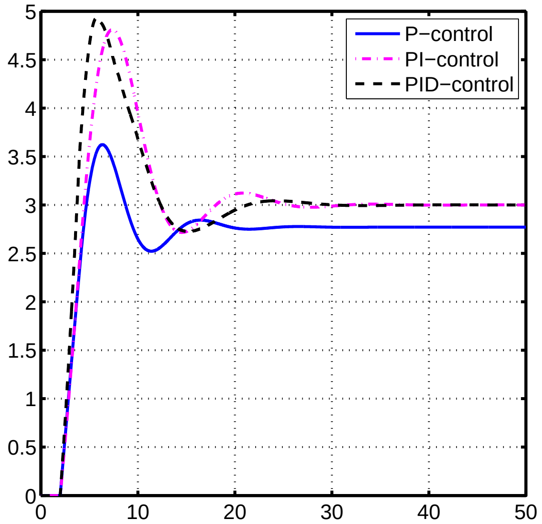
Ziegler-Nichols: Example

Computation

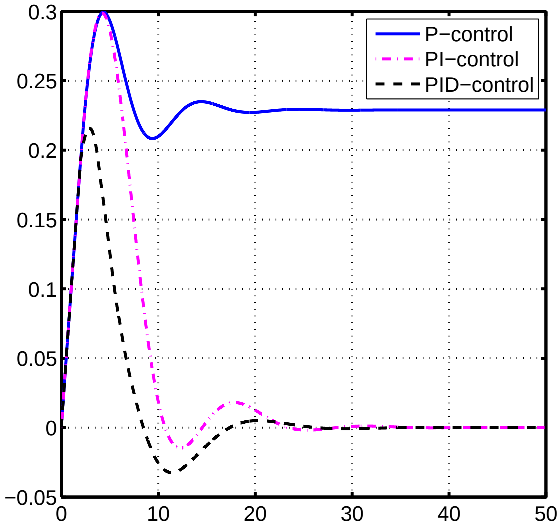
Gap 6

Ziegler-Nichols: Oscillation Method

Reference Step Response



Disturbance Step Response



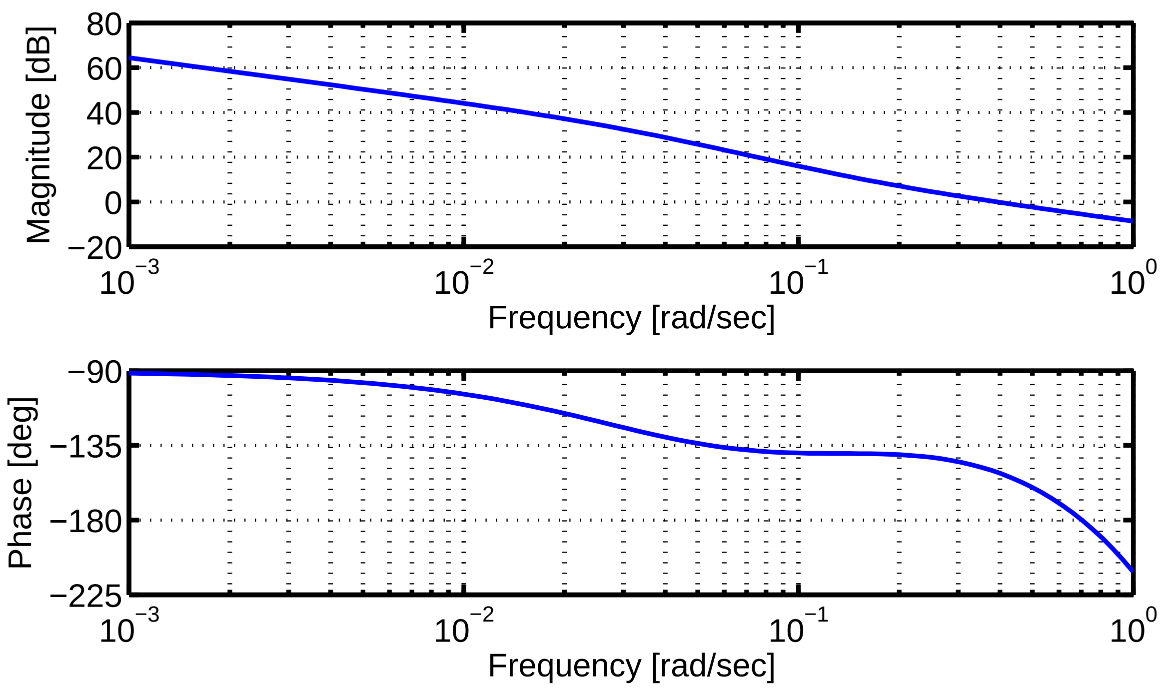
⇒ Nonzero static position error with P-control

⇒ Larger overshoot for PI and PID control due to plant delay

⇒ Similar dynamics for reference tracking and disturbance rejection

Reminder Lag Compensator Lag-Lead Compensator PID Controller Ziegler-Nichols

Ziegler-Nichols: Bode Plot



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Ziegler-Nichols: Example

Computation

Gap 7

Remarks

- Parameter computation for stability and sufficient phase margin
- Bandwidth of the closed loop according to gain crossover frequency
- Slope around the gain crossover frequency is $-20\,\mathrm{dB}$
- Further tuning of parameters is possible for example with Bode plot

Ziegler-Nichols: Reaction Curve Method

Assumption

- Stable, non-oscillatory plant: $G(s) = K \dfrac{e^{-s\tau}}{(1+sT_1)\cdots(1+sT_n)}$
  excluding $G(s) = \dfrac{K}{1+sT_1}$
- Note: plant is not modeled!

Practical Experiment

- Approach desired set-point
- Apply "small" step input
- Record plant output: process reaction curve

Step Response in Open Loop

Gap 8

Ziegler-Nichols: Reaction Curve Method

Characteristic Plant Parameters

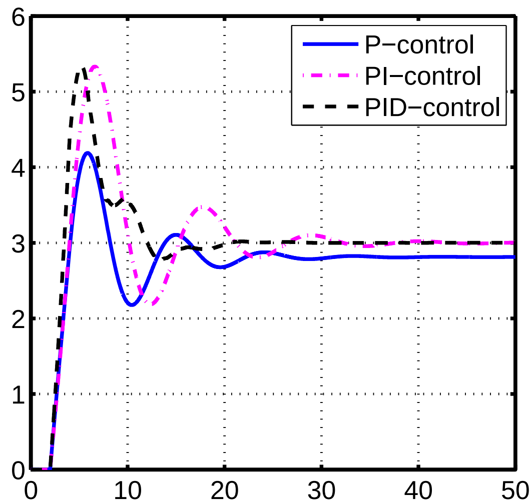
Gap 9

PID-controller Parameters

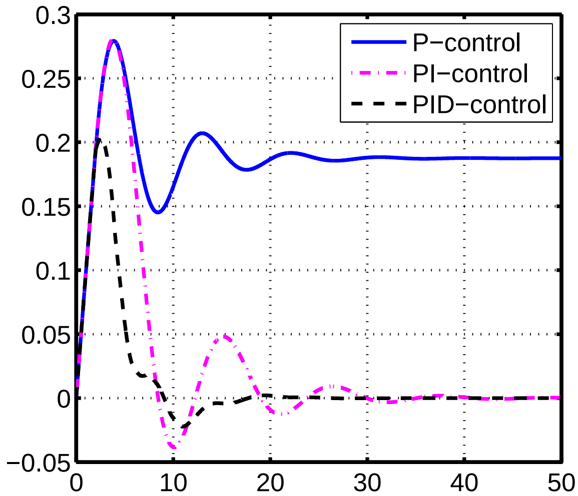
Controller	K_p	T_I	T_D
P-	$1/K \cdot T/\tau$	∞	0
PI-	$0.9/K \cdot T/\tau$	3.33τ	0
PID-	$1.2/K \cdot T/\tau$	2τ	0.5τ

Ziegler-Nichols: Oscillation Method

Reference Step Response



Disturbance Step Response



$\Rightarrow$ Similar behavior to Ziegler-Nichols Oscillation Method