ECE 488 – Automatic Control

Observability and Separation

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Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Weeks

- Plant modeling
- Properties of transfer functions
- Stability and performance
- Feedback control
- Root locus method
- Nyquist criterion and bode plot
- Lead/lag compensator and PID-controller
- Controllability and State feedback control

This week

- Observability
- Separation

Observability: Definition

State-space Model

$$\dot{x}(t) = A\,x(t) + b\,u(t)$$
$$y(t) = c^T\,x(t) + d\,u(t)$$

Observability Definition

A linear system is called completely observable if any initial state $x(0)$ at time $t = 0$ can be uniquely determined from the output signal $y(t)$ and the input signal $u(t)$ in a pre-specified time interval $0 \leq t \leq t_T$.

Gap 1

Observability: Verification

Observability Test by Kalman

A linear system of order n is completely observable if and only if the observability matrix $\mathcal{O} = \begin{bmatrix} c^T \\ c^T A \\ \vdots \\ c^T A^{n-1} \end{bmatrix}$ *has full rank n*

Example

Gap 2

Luenberg Observer: Block Diagram

Diagram

Gap 3

Luenberg Observer: Idea

State Reconstruction

- Parallel model of the linear system (simulated)
- Comparison of measured and simulated output
- Feedback to compensate error

Parallel Model

$$\dot{\hat{x}}(t) = A\,\hat{x}(t) + b\,u(t) + l(\hat{y}(t) - y(t))$$
$$\hat{y}(t) = c^T\,\hat{x}(t) + d\,u(t)$$

Variables and Parameters

- Simulated state $\hat{x}$
- Simulated output $\hat{y}$
- Feedback vector l

Luenberg Observer: Computation

State Error Computation

Gap 4

Result

$$\dot{e}(t) = \dot{\hat{x}}(t) - \dot{x}(t) = (A + l\,c^T)e(t)$$

- Stabilize observer by moving the eigenvalues of $A + l\,c^T$ in the OLHP
- Desired characteristic polynomial of $A + l\,c^T$: $p(s)$
  $\rightarrow$ We want $\det(sI - A - l\,c^T) = p(s)$

Luenberg Observer: Example

Computation

Gap 5

Separation: Combination of State Feedback and Observer

Block Diagram

Gap 6

Separation: Explanation

State Estimation

- Use the estimated state $\hat{x}$ for state feedback
  $\Rightarrow$ Controller consists of observer (for state estimation), state feedback (for pole placement) and pre-filter (for steady-state value)
- Separation principle: Closed loop eigenvalues are eigenvalues of observer plus eigenvalues of state feedback
  $\Rightarrow$ We can design state feedback and observer independently