ECE 488 – Automatic Control

Steady-State and Transient Response of LTI Systems

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Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Topics

- Linear system modeling
- LTI ODEs, block diagrams, transfer functions, state space models
- Block diagram simplification
- Nonlinear models and set-point linearization
- Analysis of Transfer Functions and Stability

This Week

- Steady-state response and Transient Response
- Dominant Poles
- Transfer Function Zeros
- Performance Specifications

Responses: Separation

Transient Response

Response of a system to an input signal for a short time period after the application of the input signal: $y_{tr}(t)$

Steady-State Response

Long term response of a system to an input signal after the transient response vanishes: $y_{ss}(t)$

Separation

$$y(t) = y_{tr}(t) + y_{ss}(t)$$

Remark

If stability of a control system is ensured, it is desired to shape the transient response of the control system

Responses: Example

Computation

Gap 1

Responses: Properties

Properties

- If $G(s)$ is BIBO stable, then the transient response converges to zero

$$\lim_{t\to\infty} y_{tr}(t) = 0$$

- If $G(s)$ is instable, then the transient response diverges

$$\lim_{t\to\infty} |y_{tr}(t)| = \infty$$

- Steady state response for LTI systems is determined by the input

Example

Gap 2

Responses: Example

Computation

Gap 3

Properties of the Transient Response: Distinct Poles

Step Response

$$Y(s) = G(s)\,\frac{1}{s} = \frac{(s-z_1)\,(s-z_2)\,\cdots\,(s-z_m)}{(s-p_1)\,(s-p_2)\,\cdots(s-p_n)}\,\frac{1}{s}$$

Decomposition for Distinct Poles

Gap 4

$$\Rightarrow Y(s) = \frac{r_0}{s} + \frac{r_1}{s-p_1} + \frac{r_2}{s-p_2} + \cdots + \frac{r_n}{s-p_n}$$

$\rightarrow p_1, p_2, \ldots, p_n$ are called the modes of the system

Properties of the Transient Response: Example

Computation

Gap 5

Properties of the Transient Response: Cases

Real Pole $p_i \neq 0$

$$\frac{r_i}{s - p_i} \bullet\!\!-\!\!\circ\; r_i\, e^{p_i\, t}\sigma(t)$$

$$\Rightarrow \lim_{t\to\infty} r_i\, e^{p_i\, t}\sigma(t) = \begin{cases} 0 & \text{if } p_i < 0 \\ \infty & \text{if } p_i > 0 \end{cases}$$

Comparison $p_i < p_j < 0$ **and** $a_i \approx a_j$

$$r_i\, e^{p_i\, t}\sigma(t) < r_j\, e^{p_j\, t}\sigma(t)$$

$\Rightarrow p_j$ dominates p_i

Comparison p_i, p_j **but** $r_j << r_i$

$$r_j\, e^{p_j\, t}\sigma(t) < r_i\, e^{p_i\, t}\sigma(t)$$

$\Rightarrow p_i$ dominates p_j

Illustration

Gap 6

Properties of the Transient Response: Example

Output Response

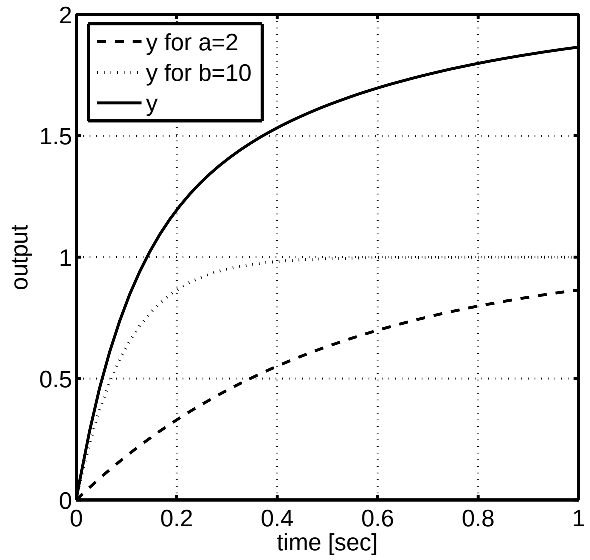
$$Y(s) = \frac{A/a + B/b}{s} - \frac{A/a}{s+a} - \frac{B/b}{s+b}$$

Computation

Gap 7

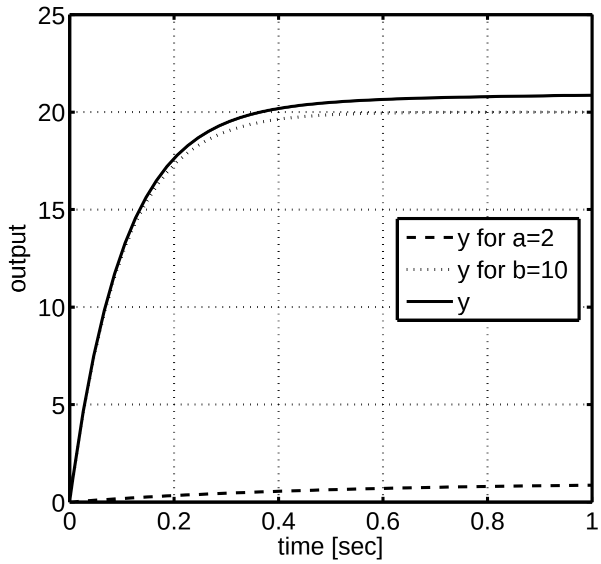
Properties of the Transient Response: Example

$A/a = B/b$



$\Rightarrow$ pole at a dominates pole at b

$A/a << B/b$



$\Rightarrow$ pole at b dominates pole at a

Properties of the Transient Response: Cases

Conjugated Complex Poles

$$\frac{r_i}{s-p_i} + \frac{r_i^\star}{s-p_i^\star} \;\bullet\!\!-\!\!\circ\; 2\,|r_i|\, e^{\mathrm{Re}(p_i)\,t} \cos(\mathrm{Im}(p_i)\,t + \angle(r_i))$$

Computation

Gap 8

Properties of the Transient Response: Cases

Properties

$$\lim_{t\to\infty} 2\,|r_i|\, e^{\mathrm{Re}(p_i)\,t} \cos(\mathrm{Im}(p_i)\,t + \angle(r_i)) = \begin{cases} 0 & \text{if } \mathrm{Re}(p_i) < 0 \\ \infty & \text{if } \mathrm{Re}(p_i) > 0 \end{cases}$$

$\Rightarrow$ exponential decay/increase similar to real pole

$$D = \frac{-\mathrm{Re}(p_i)}{|p_i|} \text{ (damping)}$$

Computation

Gap 9

Dominant Poles: Relation

Conditions for Poles

- If there is an instable pole, it dominates all stable poles
- Usually, stable poles close to the imaginary axis (slow convergence) dominate stable poles far from the imaginary axis (fast convergence)
- Exceptions exist depending on the residues of the modes

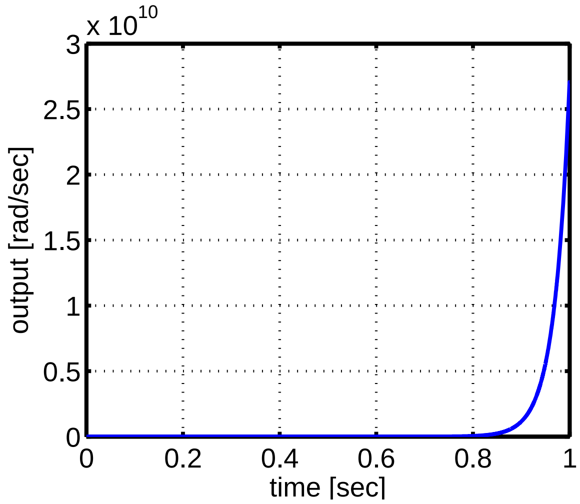
Examples

Gap 10

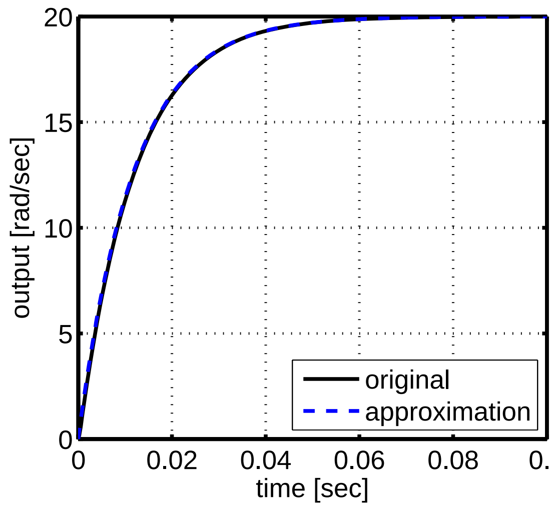
Dominant Poles: Simulation

Gap 11

Magnetic Suspension



DC Motor



Plant Zeros: Basics

Stable Plant

$$G(s) = \frac{(s - z_1)\cdots(s - z_m)}{(s - p_1)\cdots(s - p_n)}$$

Minimum-Phase Zero

- Zeros in the open left half plane: $\mathrm{Re}(z_j) < 0$

Non-minimum Phase Zero

- Zero in the right half plane: $\mathrm{Re}(z_j) > 0$

Example

Gap 12

Plant Zeros: Minimum Phase Zeros and Dominant Poles

Suppression of Dominant Poles

- Assume a zero z_j is close to the dominant pole p_i: $z_j \approx p_i$

$$G(s) = \frac{(s - z_j)\tilde{B}(s)}{(s - p_i)\tilde{A}(s)}$$

- Residue of mode p for step response

$$r_i = \lim_{s \to p_i} \frac{G(s)\,(s - p_i)}{s} = \lim_{s \to p_i} \frac{(p_i - z_j)\tilde{B}(p_i)\,(s - p_i)}{(s - p_i)\tilde{A}(p_i)} \approx 0$$

$\Rightarrow$ Dominant mode with pole p_i does not appear in the step response
$\Rightarrow$ If $z_j \approx p_i$, dominant modes can be suppressed and other modes become dominant

Plant Zeros: Minimum Phase Zeros and Dominant Poles

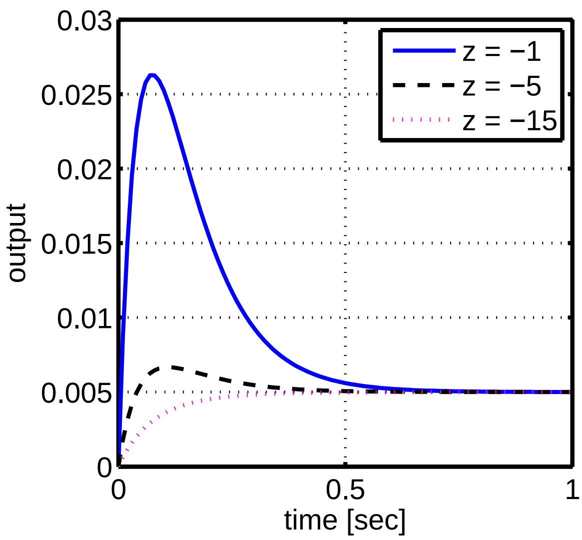
Overshoot

- Dominant plant pole at p with $\mathrm{Re}(p) < 0$
- Slow minimum phase plant zero z with $\mathrm{Re}(p) << \mathrm{Re}(z) < 0$

$\Rightarrow$ Overshoot of the step response

Example Transfer Function

$$G(s) = \frac{-1}{z}\frac{(s - z)}{(s + 10)(s + 20)}$$



Plant Zeros: Non-minimum Phase

Statement

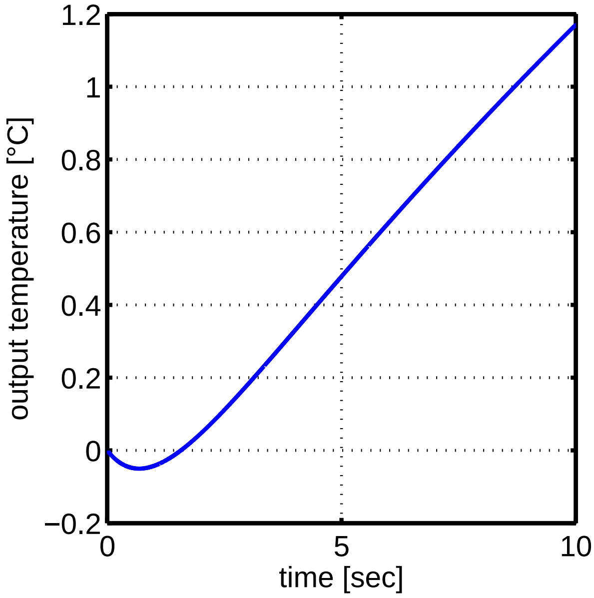
- k non-minimum phase zeros in $G(s)$
  $\Rightarrow$ Step response intersects with time-axis k times
  $\Rightarrow$ Undershoot whenever there are non-minimum phase zeros

Example

$$G(s) = \frac{5(s-1)}{(s+1)(1+30s)}$$

$\Rightarrow$ One intersection with time axis

Example Simulation



Performance Specifications: Step Response Characteristics

Performance Specifications

- Describe desired behavior of a control system based on step response
- Define performance metrics that can be used in practice

Illustration

Gap 13

Performance Specifications: Step Response Characteristics

Steady State Value

- Output in steady state: $y_\infty = \lim_{t\to\infty} y(t)$

Rise Time

- Quantifies speed of response: first time t_r such that $y(t_r) = 0.95 y_\infty$

(Percent) Overshoot

- Quantifies damping of response: $M_p = \max_{t\in\mathbb{R}} \frac{y(t) - y_\infty}{y_\infty}$

Peak Time

- Time until first peak of overshoot is reached: t_p

Settling Time

- Quantifies how long it takes until response stays around the final value (for example 2% or 5%): t_s

Performance Specifications: Example

First-order Lag

$$Y(s) = \frac{K}{1 + s\,T}\,\frac{1}{s} \;\bullet\!\!-\!\!\circ\; y(t) = \sigma(t)(1 - e^{-t/T})$$

Computation

Gap 14

Performance Specifications: Example

Second-order Lag ($D < 1$)

$$Y(s) = \frac{K}{1 + 2\,D\,T\,s + T^2\,s^2}\,\frac{1}{s} \quad \bullet\!\!-\!\!\!-\!\!\circ$$

$$y(t) = \sigma(t) K \big(1 - \frac{e^{-Dt/T}}{\sqrt{1-D^2}} \sin(\frac{\sqrt{1-D^2}}{T} t + \arctan \frac{\sqrt{1-D^2}}{D})\big)$$

Computation

Gap 15

Performance Specifications: Example

Computation

Gap 16